the pattern reverse with a frequency variation within 10 per cent. It is not necessary to use the ferrite below and above ferromagmagnetic resonance to achieve this reverse.

Fig. 4 shows the graphs of decoupling and forward loss of the two ports adjacent to the input for two typical cases. For a properly shaped ferroxcube 5D3 ($4\pi M_s \approx 2900$ gauss) a large separation of the two optimum frequencies for opposite circulation has been achieved whereas for ferroxcube 5A2 ($4\pi M_s \approx 1450$ gauss) with another shape and in another magnetic field the oppositely circulating frequencies have been brought close together. The cross decoupling was in all cases observed to be more than 15 db. It should be remarked that for the cross decoupling much higher db-values can be achieved if more attention is paid to the impedance matching.

Figs. 5 and 6 give the optimum working frequencies and the pertaining ratios of decoupling and forward losses as a function of the applied magnetic field and the height of the ferrite body, respectively. Fig. 7 gives the separation of the optimum frequencies for both senses of circulation for three different saturation magnetizations. Shape, applied field, and matching element are kept constant.

Finally it may be concluded that the described device—apart from some technical imperfections such as the poor cross decoupling—can be used as a quadruplexer if the power of the transmitters is not very much higher than that of the received signals. A proper design, such as used in the experiments described above, has two main advantages; namely, a normal polycrystalline ferrite can be used and the separation of the two oppositely circulating frequencies can be varied over a rather wide range because no absorption losses are limiting it.

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A Resonant Slope Amplifier Using A Microwave Pump Frequency*

Resonant slope (or resonant dielectric) amplifiers are of interest because of their high-input impedance and low-frequency capability.

The desire for a resonant slope amplifier, and the ready availability of microwave components in the Boeing Applied Physics Laboratory, led to the development of such an amplifier using a microwave pump frequency.

Fig. 1 shows a sketch of the amplifier system. The only nonstandard component used in the system is the cavity; the cavity is a section of X-band waveguide terminated at one end by an adjustable short, and at the other (input-output end) by a coupling iris. An MA 4296 varactor is mounted in the cavity at a position where the electric field is maximum (the cavity operates in the TE_{102} mode). Actually, a standard tunable detector mount such as the HPX485B could be used, with a coupling iris, for the cavity if the crystal mount were altered to provide a means for biasing the varactor.

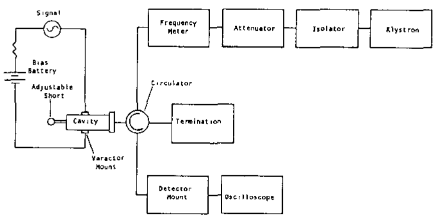

Fig. 1—Sketch of resonant slope amplifier system.

The principle of operation is virtually identical to that of the conventional lumped-circuit resonant slope amplifier. The varactor and the cavity form the resonant circuit. With no signal input, the adjustable short position and the klystron frequency are adjusted so that the circuit is slightly off resonance—the no-signal operating point is on the slope of the resonance curve. (A circulator is used, therefore resonance is indicated by a dip in the output; thus, the response curve of the circuit and circulator is inverted as compared to the response curve of the lumped, tuned circuit usually used.) The varactor is reverse-biased, and the signal is applied in series with the bias.

The signal varies the varactor capacitance which, in turn, varies the resonant frequency (or operating point) of the circuit. Thus, for small signals, the amplitude of the energy which is reflected by the cavity, and thus appears in the output of the circulator, follows that of the input signal. And, under proper conditions, the amplitude of the detected output signal can be much greater than that of the input.[1-4]

The power gain of our experimental amplifier at signal frequencies from dc through about 50 kc was constant at approximately 42 db. The pump frequency was 8.5 Gc. The input impedance, which is mainly a function of the varactor used, was about 2 megohms; impedances as high as 10^{10} ohms appear feasible.[5] The output impedance was about 1800 ohms.

The obvious disadvantage of the microwave version of the dielectric amplifier is its bulk. However, it is easily assembled from standard microwave components and comparatively large voltage gains are possible because of the high-Q microwave cavity.

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A Three-Port Network with Constant Phase Difference Properties*

I. Introduction

The 90° hybrid and the 180° TEM magic tee can, at least theoretically, be made to work over bandwidths of 10 to 1, and more. The device described is not limited to 90° or 180°. It provides a phase shift between its outputs that is not only constant with frequency, but also capable of being set at any value. In addition, the ratio of the powers delivered to the outputs may be set as desired.

II. Polarizing Element

There are classes of antennas in existence that have constant properties over very wide frequency ranges. One of the principal types is the arithmetic spiral. The arithmetic spiral is a circularly polarized element made up of two conductors winding in a flat plane (Fig. 1). The element is usually fed by

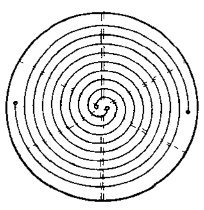

Fig. 1—The arithmetic spiral.

means of a coaxial line connected between the two conductors at their center terminals. The bandwidth of the spiral is limited at the high end by the difficulty in winding the small central turns. No such difficulty is encountered at the low end, although in practice the circumference of the outer turns is kept less than 2λ at the highest operating frequency. Bandwidths of four to one and better are easily obtained with this type of element. The device described utilizes the wide-band polarization properties of arithmetic spirals to provide a constant phase difference between the outputs of a three-port network.[1] The network is frequency-

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[1] The arithmetic spiral is used in this discussion by way of illustration. Actually, any broad-band circularly polarized element may be used.

independent to the same extent as the spirals used. The phase difference remains exactly constant at all frequencies where their axial ratios are equal to one.

III. Theory of Operation

The fact that the phase of a circularly polarized antenna is dependent upon its rotation has been successfully used by Brown and Dodson in developing a novel antenna design. This rotational phase interdependence is the underlying principle in the network under discussion.

Fig. 2 is a general representation of the network. Energy enters the tee junction from port 1 and splits between arms 2 and 3. P_1 and P_2 are polarizers (such as arithmetic spirals) which convert the input to the circularly polarized mode. The polarizers transfer the energy through equal lengths of transmission line to ports 2 and 3.

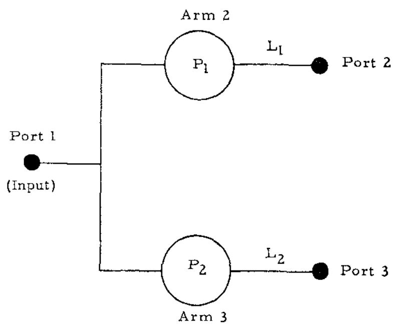

Fig. 2—A three-port network with constant phase difference properties.

Let us consider the ideal case where both polarizers are identical and always provide axial ratios equal to one. Assume that the polarizers initially have the same rotation with respect to an arbitrary fixed axis; *i.e.*, have the same orientation in space. Since arms 2 and 3 are identical in every way, there will be no phase difference at their outputs. However, if one of the polarizers is rotated with respect to the other, a phase difference will appear at ports 2 and 3. This phase difference is independent of frequency and is numerically equal to the angle of rotation. Symbolically, if the polarizers were originally at an angle α_1 with respect to the x axis and one of them, say P_2, were rotated to an angle α_2, then the phase difference between ports 2 and 3 is

$$\phi_{23} = \alpha_2 - \alpha_1. \tag{1}$$

Eq. (1) may be proved as follows: Consider the x and y components of a circularly polarized wave E_1,

$$E_{1x} = E \cos \omega \left(t - \frac{z}{v} \right)$$

$$E_{1y} = E \cos \left[\omega \left(t - \frac{z}{v} \right) - \frac{\pi}{2} \right]$$

where

z = the direction of propagation
v = the velocity of the wave.

In the plane $z = 0$ these equations reduce to

$$E_{1x} = E \cos \omega t$$

$$E_{1y} = E \cos \left(\omega t - \frac{\pi}{2} \right) = E \sin \omega t.$$

At any time t_1 the angle between the resultant electric vector and the x axis is

$$\alpha_1 = \tan^{-1} \frac{E_{1y}}{E_{1x}} = \tan^{-1} \frac{E \sin \omega t_1}{E \cos \omega t_1}$$

$$\alpha_1 = \omega t_1. \tag{2}$$

Now let us consider the x and y components of a second wave E_2. E_2 is identical to E_1 in every way except that it is shifted in phase by a constant amount, ϕ. At $z = 0$,

$$E_{2x} = E \cos (\omega t + \phi)$$

$$E_{2y} = E \cos \left(\omega t - \frac{\pi}{2} + \phi \right) = E \sin (\pi t + \phi).$$

The angle of the electric vector at the same time t_1 is, in this case,

$$\alpha_2 = \tan^{-1} \frac{E_{2y}}{E_{2x}} = \tan^{-1} \frac{E \sin (\omega t_1 + \phi)}{E \cos (\omega t_1 + \phi)}$$

$$\alpha_2 = \omega t_1 + \phi. \tag{3}$$

The fact that $\alpha_2 \neq \alpha_1$ means that one wave is rotated with respect to the other; *i.e.*, one polarizer is rotated with respect to the other. Taking the angular difference,

$$\alpha_2 - \alpha_1 = \omega t_1 + \phi - \omega t_1$$

$$\alpha_2 - \alpha_1 = \phi,$$

which proves (1).

IV. The Network in Practice

In practice, (1) is true only for perfect circular polarization. However, slight errors due to small ellipticities should not destroy its usefulness.

The fact that the outputs of arms 2 and 3 are circularly polarized may be less convenient than if they were coaxial lines. It would be useful then to terminate arms 2 and 3 with depolarizers which reconvert the circularly polarized mode. If these depolarizers are identical and have the same rotation, none of the properties discussed will be affected.

In any configuration such as the above at least one serious problem is to be expected, that of transducing from the polarizing elements to the intervening transmission line. If spiral elements are used, the problems of higher-order mode excitation and proximity of the transmission line walls to the spiral conductors will have to be solved before any useful component can be built.

V. Effect of Unequal Power Split

The amplitudes at ports 2 and 3 are completely independent of the phase relationship. This is borne out by (2) and (3) which show that the angle the electric vector makes with the x axis is a function of frequency, time, and initial phase only.

VI. Phase Difference Variation

Fig. 3 is a drawing of what a practical network might look like. The coaxial tee feeds two spirals backed by conical cavities. They, in turn, direct their energy down cylindrical waveguides which are terminated in two additional spirals. Rotary joints before and after one of the polarizers allow rotation. Finally, the output appears at a pair of coaxial connectors. The numerical value of the phase difference may be simply controlled by rotating the polarizer sandwiched between the rotary joints to the desired angle.

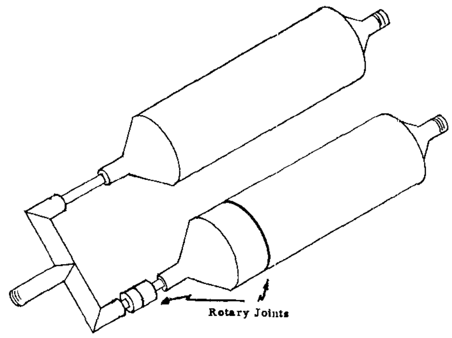

Fig. 3—Example of a practical network.

VII. Conclusion

A network has been described which is theoretically frequency-independent. In practice, however, several limitations crop up. Among these are the bandwidth of the spiral elements, the bandwidth of the waveguide used, and spiral ellipticity.

The most serious design problem to be expected is the transformation from the spiral elements to the circular waveguide.

It is hoped that a network will be simulated in the near future which will adequately test these limitations.

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A Simple Formula for Calculating Approximate Values of the First Zeros of a Combination Bessel Function Equation*

The solution of many problems in microwave theory, particularly those relating to waveguides having curved boundary surfaces, is dependent upon a determination of the zeros of the Bessel function equation

$$J_p(x) N_p(kx) - J_p(kx) N_p(x) = 0 \tag{1}$$

where J_p and N_p are respectively the Bessel functions of the first and second kinds, of order p. In a majority of the cases arising in waveguide theory, the parameters k and p

* Received September 28, 1962.